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Searching for the Credit Portfolio Structure and Building Portrait of Prospective Borrower

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Abstract: Article describes business process engineering of a commercial bank in part of developing credit policy. Proposed model gives tools of finding optimal structure of bank's credit portfolio and finding optimal financial performance indicators of a potential borrower using differential equations and modeling of stochastic variables and criterion of indifference of a bank. Finally, model forms the base and rules for the simulation of performance indicators of potential borrowers using Monte-Carlo method for future time instances.

Keywords: Differential equations, stochastic variables, business process engineering, bank portfolio modeling, probability of default, mean reversion process, credit policy, indifference criterion of a bank.

1. INTRODUCTION

Currently, commercial banks are faced with the problem of determining the financial indicators of the sample borrower. Potential borrowers are chosen, usually by analysing one indicator - the debt/EBITDA ratio of the company that is not always correct and complete. A deeper analysis of the characteristics of the borrower is not carried out that complicates the process of making a decision on bank lending. This increases the risk of both: a borrower who is waiting for loan and the bank because it may lose the borrower. Additionally, the lack of credit policy definition of prospective borrower results in appearance services such as bank underwriting, which in fact is caring out the same job as credit analyst, thereby increasing bank costs and increasing the timeline of consideration of customers applications. Another point is that the majority of banks assess potential borrowers on the basis of the already known performance indicators that reflect the results of prior periods. It is much more important, what result the company will show after receiving the loan. The proposed model forms the base and rules for the simulation of performance indicators of borrowers for future periods of time (from the quarter to the year in advance). For each borrower numerical characteristics of the simulated distribution of financial indicators are calculated. Then calculation of the expected probability of default of each borrower for future time instants is carried out. Finally, weighted average expected probability of default on the loan portfolio is calculated, as well as target default probability of the portfolio with new borrowers is found out. Using the criterion of indifference for the bank, which is provided in part four, and target probability of default allows choosing possible linear combinations of financial indicators of potential borrowers. That is the final aim of the proposed approach.

2. CONTINUOUS-TIME MODEL OF MAIN INDICATORS OF A COMPANY

2.1 Main company's indicators for determination of default probability

Determination of probability of default is based on modelling following indicators of a company at the time instance t: Working capital/Assets, (Assets-Working capital)/Assets, Debt/Assets, Working Capital Turnover, Net Debt/EBITDA. These coefficients are defined by the following system of equations:

$$\begin{cases} EBITDA(t) = R(t) - Cost(t); \\ dD(t) = (\alpha(t) - X(t))dt \\ dX(t) = (Y(t) + Dep(t) - Capx(t) - WC_{need}(t))dt \\ WC(t) = R(t)C_{WC} \\ Capx(t) = Ma\,\mathrm{int}(t) + InvCapx \\ Y(t) = (R(t) - Cost(t) - Dep(t) - Percent(t))(1-\tau) \\ A(t) = PPE(t) + WC(t) + \alpha \\ Percent(t) = r(t)D(t). \end{cases} \quad (1)$$

Here:

EBITDA(t) is earning before interest, tax, depreciation and amortization of a company;

D(t) is debt of a company;

WC(t) is working capital of a company;

Capx(t) is capital expenditure of a company;

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*Ma*int(t) is maintenance expenses of a company;

*Inv*Capx(t) is investment expenses of a company;

R(t) is revenue;

Cost(t) is total cost of company;

Y(t) is net profit after tax;

PPE(t) is property, plant and equipment;

A(t) is total assets of a company;

X(t) is cash flow;

r(t) is average interest rate;

Percent(t) are financial expenses of a company.

2.2 Continuous Time Model of Profit and Loss Indicators

Revenue dynamics is described by the stochastic differential equation:

$$\frac{dR(t)}{R(t)} = \mu(t)dt + \sigma(t)dz. \tag{2}$$

Here: μ (t), drift, is expected revenue growth, dz is Wiener process, σ (t) is the volatility or anticipated changes of revenue. Expected revenue growth depends on two parameters: μ_{capex} – expected growth rate, connected with maintenance capital expenditures, μ_g – expected growth rate except maintenance capital expenditures:

$$\mu(t) = \mu_g(t) + \mu_{CAPEX}(t) \tag{3}$$

The anticipated changes in revenues are tending back to the average long-term volatility:

$$d\sigma(t) = k_1(\overline{\sigma} - \sigma(t))dt. \tag{4}$$

Here k_1 is mean reversion coefficient. Expected growth rate except maintenance is described as follows:

$$d\mu_g(t) = k_\mu(\overline{\mu_g} - \mu_g(t))dt + \eta_g(t)dz_g. \tag{5}$$

Process which is defined in (5) is Ornstein-Uhlenbeck mean reversion process with anticipated changes process η_g.

$$d\eta_g(t) = k_2(\overline{\eta_g} - \eta_g(t))dt. \tag{6}$$

Expected growth rate, connected with maintenance is described by following equation:

$$\begin{cases} \mu_{CAPEX}(t) = CapMa\ \text{int}(t)\psi(t) \\ CapMa\ \text{int}(t)dt = \dfrac{dMa\ \text{int}(t)}{Ma\ \text{int}(t)} \\ d\psi(t) = k_\psi(\overline{\psi} - \psi(t))dt + \lambda(t)dz_\psi. \end{cases} \tag{7}$$

Here ψ shows the relationship between dynamics of maintenance expenditures *Ma*int(t) and revenue growth rate. Anticipated changes in process above are also defined deterministically by mean reversion process:

$$\lambda(t) = k_3(\overline{\lambda} - \lambda(t))dt. \tag{8}$$

Total costs at any moment depend on two components. They are fix costs and component which is dependent on revenues:

$$Cost(t) = \gamma(t)R(t) + FC(t). \tag{9}$$

Variable cost parameter γ(t) is cost function and consists of uncertainty about market share, competitors and is described by stochastic equation:

$$d\gamma(t) = k_4(\overline{\gamma} - \gamma(t))dt + \varphi(t)dz_2. \tag{10}$$

Here volatility is also a mean reversion process:

$$d\varphi(t) = k_5(\overline{\varphi} - \varphi(t))dt. \tag{11}$$

In this way, net profit after tax of a company equals:

$$Y(t) = (R(t) - Cost(t) - Dep(t) - Percent(t))(1-\tau). \tag{12}$$

Here Dep(t) is accumulated depreciation in period t, Percent(t) is percent expenses and τ is effective corporate tax rate. Percent expenses are described by following equation:

$$Percent(t) = r(t)D(t). \tag{13}$$

Here D(t) is debt of a company and r(t) is interest rate, cost of debt. The interest rate depends on three components: the cost of bank's funding (S), basic margin of a bank (M) and the risk premium (G, depends on the company's probability of default - PD):

$$r(t) = S(t) + M(t) + G(t, PD). \tag{14}$$

The dynamics of the cost of a bank funding is described by stochastic process:

$$\begin{aligned} dS(t) &= k_S(\overline{S} - S(t))dt + \varpi(t)dz_S \\ d\varpi(t) &= k_\varpi(\overline{\varpi} - \varpi(t))dt. \end{aligned} \tag{15}$$

2.3 Continuous Time Model of Balance Sheet Components Indicators

Property plant and equipment, working capital and debt dynamics is described here. Property plant and equipment dynamics depends on capital expenditures and depreciation:

$$dPPE(t) = (Capx(t) - Dep(t))dt. \tag{16}$$

Depreciation is fixed proportion DR of fixed assets:

$$Dep(t) = DR * PPE(t). \tag{17}$$

Capital expenditures are defined by the following equation:

$$Capx(t) = Ma\ \text{int}(t) + InvCapx(t). \tag{18}$$

Here *Ma*int(t) is maintenance expenditures, which support the activities of a company at the current level, *Inv*Capx is investment in new projects. An order of maintenance expenses is often the same as depreciation with some anticipated fluctuations:

$$dMa\,\mathrm{int}(t) = k_{Ma\,\mathrm{int}}(Dep(t) - Ma\,\mathrm{int}(t))dt + \nu(t)dz_m \qquad (19)$$
$$d\nu(t) = k_\nu(\overline{\nu} - \nu(t))dt.$$

Investments in new projects are assumed to be on a historical level. Another possibility is to take into account company's plan about investments in new projects. According to problem statement, the company can borrow funds for two purposes: financing of working capital and financing of capital expenditures. The dynamics of working capital (working capital need) is described as follows:

$$WC(t) = R(t)C_{WC} \qquad (20)$$
$$WC_{need}\,dt = dR(t)C_{WC}.$$

Where C_{WC} is working capital turnover (the ratio of the length of the cycle of working capital to a period of time). Working capital turnover is considered to be a stochastic mean reversion process:

$$\begin{cases} dC_{WC}(t) = k_{WC}(\overline{C_{WC}} - C_{WC}(t))dt + \eta_{WC}(t)dz_{WC} \\ \eta_{WC}(t) = k_{\eta_{WC}}(\overline{\eta_{WC}} - \eta_{WC}(t))dt. \end{cases} \qquad (21)$$

Total assets of the company are calculated as follows:

$$A(t) = PPE(t) + WC(t) + \alpha(t). \qquad (22)$$

Where α is the required minimum cash balance. Taking into account previous equations, debt dynamics is described as follows:

$$dD(t) = (\alpha(t) - X(t))dt. \qquad (23)$$

Where α is the required minimum cash balance, X(t) is cash balance which is generated at time instant t:

$$dX(t) = (Y(t) + Dep(t) - Capx(t) - WC_{need}(t))dt. \qquad (24)$$

If X(t)>α(t), company pays out debt in amount X(t)-α(t). If X(t)<α(t), company increases it debt in amount α(t)-X(t).

3. DISCRETE-TIME APPROXIMATION OF MAIN INDICATORS OF A COMPANY

3.1 Construction of an Iterative Scheme for Solving Stochastic Differential Equations

This part demonstrates adduction of differential stochastic equations which are used for describing activity of a company to iterative scheme in discrete time using Ito's lemma. Complete algorithm for obtaining iterative scheme for stochastic differential equation (5) is described below. Process (5) is subject of diffusive Ito equation.

$$dF = A(x,t) + B(x,t)dW \qquad (24)$$
$$x = G(F,t).$$

Where G is inverse function to F. Ito's lemma equation is given next:

$$dF = f(t)dt + s(t)dW$$
$$\left[\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{b^2(x,t)}{2}\frac{\partial^2 F}{\partial x^2}\right] = f(t) \qquad (25)$$
$$b(x,t)\frac{\partial F}{\partial x} = s(t).$$

Compatibility conditions for searching for functions s(t), f(t), F(t) are below:

$$\begin{cases} \dfrac{\partial F}{\partial x} = \dfrac{s(t)}{b(x,t)} \\ \dfrac{\partial F}{\partial t} + s(t)\left[\dfrac{a(x,t)}{b(x,t)} - \dfrac{1}{2}\dfrac{\partial b(x,t)}{\partial x}\right] = f(t) \\ \dfrac{1}{s(t)}\dfrac{\partial}{\partial t}\left\{\dfrac{s(t)}{b(x,t)}\right\} = \dfrac{1}{2}\dfrac{\partial^2 b(x,t)}{\partial x^2} - \dfrac{\partial}{\partial x}\left\{\dfrac{a(x,t)}{b(x,t)}\right\}. \end{cases} \qquad (26)$$

Solution of stochastic equation can be obtained in the following form:

$$F(x(t),t) = F(x(t_0),t_0) + \int_{t_0}^{t} f(\tau)d\tau + \left[\int_{t_0}^{t} s(\tau)^2 d\tau\right]^{1/2}\varepsilon. \qquad (27)$$

For equation (5):

$$a(x,t) = k_\mu(\overline{\mu_g} - \mu_g) \qquad (28)$$
$$b(x,t) = \eta_g.$$

First aim is to find s(t):

$$\frac{1}{s(t)}\frac{\partial}{\partial t}\left\{\frac{s(t)}{\eta_g}\right\} = \frac{1}{2}\frac{\partial^2\sigma}{\partial {\mu_g}^2} - \frac{\partial}{\partial \mu_g}\left\{\frac{k_\mu(\overline{\mu_g} - \mu_g)}{\eta_g}\right\}$$
$$\frac{1}{\eta_g s(t)}\frac{ds(t)}{dt} = \frac{k_\mu}{\eta_g} \qquad (29)$$
$$\frac{ds}{s} = k_\mu dt$$
$$s(t) = \eta_g e^{k_\mu t}.$$

Next step is calculation f(t) and F(t):

$$\frac{dF}{d\mu_g} = e^{k_\mu t}$$
$$F(\mu_g(t),t) = \mu_g e^{k_\mu t} \qquad (30)$$
$$f(t) = k_\mu \mu_g e^{k_\mu t} + \eta_g e^{k_\mu t}\frac{k_\mu(\overline{\mu_g} - \mu_g)}{\eta_g}$$
$$f(t) = k_\mu \overline{\mu_g} e^{k_\mu t}.$$

The last step is to substitute functions F, f, s to equation (27) and express it relatively μ_g:

$$\mu_g e^{k_\mu t} = \mu_{g0} e^{k_\mu t_0} + \int_{t_0}^{t} k_\mu \overline{\mu_g} e^{k_\mu \tau} d\tau + \left[\int_{t_0}^{t} \left(\eta_g e^{k_\mu \tau}\right)^2 d\tau\right]^{1/2}\varepsilon. \qquad (31)$$

Assuming t_0=0, final iterative scheme for process (5) is:

$$\mu_g(t+\Delta t)=\mu_g(t)e^{-k_\mu \Delta t}+\overline{\mu_g}(1-e^{-k_\mu \Delta t})+\frac{\eta_g}{\sqrt{2k_\mu}}\sqrt{1-e^{-2k_\mu \Delta t}}\,\varepsilon. \quad (32)$$

3.2 Iterative Schemes for Solving Equations of Dynamics of Company's Profit and Loss Indicators

Iterative schemes for equations which describe profit and loss indicators of a company may be represented in following way. First of all dynamics of revenue is described. After that, dynamics of costs and financial expenses are calculated. Revenue dynamics is shown in following system:

$$\begin{cases}
\sigma(t)+\sigma_0 e^{-k_1 t}+\overline{\sigma}(1-e^{-k_1 t}) \\
\mu_g(t+\Delta t)=\mu_g(t)e^{-k_\mu \Delta t}+\overline{\mu_g}(1-e^{-k_\mu \Delta t})+\dfrac{\eta_g}{\sqrt{2k_\mu}}\sqrt{1-e^{-2k_\mu \Delta t}} \\
\eta_g(t)=\eta_{g0}e^{-k_2 t}+\overline{\eta_g}(1-e^{-k_2 t}) \\
\psi(t+\Delta t)=\psi(t)e^{-k_\psi \Delta t}+\overline{\psi}(1-e^{-k_\psi \Delta t})+\dfrac{\lambda}{\sqrt{2k_\psi}}\sqrt{1-e^{-2k_\psi \Delta t}}\,\varepsilon \\
\lambda(t)+\lambda_0 e^{-k_3 t}+\overline{\lambda}(1-e^{-k_3 t}) \\
CapMa\operatorname{int}(t+\Delta t)=\dfrac{1}{\Delta t}\ln\left[\dfrac{Ma\operatorname{int}(t+\Delta t)}{Ma\operatorname{int}(t)}\right] \\
\mu_{CAPEX}(t)=CapMa\operatorname{int}(t)\psi(t) \\
\mu(t)=\mu_g(t)+\mu_{CAPEX}(t) \\
R(t+\Delta t)=R(t)e^{\left\{\left[\mu(t)-\frac{\sigma^2(t)}{2}\right]\Delta t+\sigma(t)\sqrt{\Delta t}\varepsilon\right\}}.
\end{cases} \quad (33)$$

Dynamics of cost of sales is described as follows:

$$\begin{cases}
\gamma(t+\Delta t)=\gamma(t)e^{-k_4\Delta t}+\overline{\gamma}(1-e^{-k_4\Delta t})+\dfrac{\varphi}{\sqrt{2k_4}}\sqrt{1-e^{-2k_4\Delta t}}\,\varepsilon \\
\varphi(t)=\varphi_0 e^{-k_5 t}+\overline{\varphi}(1-e^{-k_5 t}) \\
Cost(t)=\gamma(t)R(t)+FC(t).
\end{cases} \quad (34)$$

Next step is to show dynamics of financial expenses:

$$\begin{cases}
S(t+\Delta t)=S(t)e^{-k_S\Delta t}+\overline{S}(1-e^{-k_S\Delta t})+\dfrac{\varpi}{\sqrt{2k_S}}\sqrt{1-e^{-2k_S\Delta t}}\,\varepsilon \\
\varpi(t)=\varpi_0 e^{-k_\varpi t}+\overline{\varpi}(1-e^{-k_\varpi t}) \\
r(t)=S(t)+M(t)+G(t,PD) \\
Percent(t)=r(t)D(t).
\end{cases} \quad (35)$$

After equations (33)-(35) are performed, the last step is to calculate net profit after tax as it is described in equation (12).

3.3 Iterative Schemes for Solving Equations of Dynamics of Company's Balance Sheet Indicators

Iterative schemes for equations which describe balance sheet indicators of a company may be represented in following way:

$$\begin{cases}
\nu(t)=\nu_0 e^{-k_\nu t}+\overline{\nu}(1-e^{-k_\nu t}) \\
Ma\operatorname{int}(t+\Delta t)=Ma\operatorname{int}(t)e^{-k_{Maint}\Delta t}+\overline{Ma\operatorname{int}}(1-e^{-k_{Maint}\Delta t})+\dfrac{\nu}{\sqrt{2k_{Maint}}}\sqrt{1-e^{-2k_{Maint}\Delta t}}\,\varepsilon \\
Capx(t)=Ma\operatorname{int}(t)+InvCapx(t) \\
PPE(t+\Delta t)=PPE(t)+(Capx(t+\Delta t)-Dep(t+\Delta t))\Delta t \\
\eta_{WC}(t)=\eta_{WC0}e^{-k_{\eta WC}t}+\overline{\eta_{WC}}(1-e^{-k_{\eta WC}t}) \\
C_{WC}(t+\Delta t)=C_{WC}(t)e^{-k_{WCt}\Delta t}+\overline{C_{WC}}(1-e^{-k_{WC}\Delta t})+\dfrac{\eta_{WC}}{\sqrt{2k_{WCt}}}\sqrt{1-e^{-2k_{WCt}\Delta t}}\,\varepsilon \\
WC_{need}(t+\Delta t)=C_{WC}\dfrac{R(t+\Delta t)-R(t)}{\Delta t} \\
X(t+\Delta t)=X(t)+(Y(t+\Delta t)+Dep(t+\Delta t))-Capx(t+\Delta t))-WC_{need}(t+\Delta t))\Delta t \\
D(t+\Delta t)=D(t)+(\alpha(t+\Delta t)-X(t+\Delta t))\Delta t.
\end{cases} \quad (36)$$

These equations are used to calculate company's performance indicators for computing probability of default.

3. PROBABILITY OF DEFAULT MODELING

Logistic model is adopted as the base model for calculating the probability of default. It is based on five factors which are coefficients, calculated on the basis of the financial performance of the borrower.

$$PD=\frac{e^\theta}{1+e^\theta} \quad (37)$$
$$\theta = -1{,}122-0.272x_1-4.731x_2+5.702x_3+0.002x_4+0.119x_5.$$

Here x_i is Working capital/Assets, x_2 is (Assets-Working capital)/Assets, x_3 is Debt/Assets, x_4 is Working Capital Turnover, x_5 is Net Debt/EBITDA.

4. INDIFFERENCE CONDITION FOR A BANK

Assuming that borrower has probability of default PD, there are two possible outcomes for a bank: first – borrower will pay back the loan and interest on the debt with probability (1-PD), second – the borrower will declare a default and the bank will receive a discount value of the collateral, less selling costs with probability PD. Very important assumption of the model is that defaults are not correlated. Thus, the condition of indifference of borrower default for a bank can be written as follows:

$$PD\left[disc(t)PLEDGE(t)-SC(t)\right]e^{-r(T-t)}=(1-PD)\left[V+Percent\right]. \quad (38)$$

Here disc(t) is pledge discount, SC is selling cost, T is moment of selling of pledge, V is liability of the company in a bank. If bank's policy assumes definite insecurity, the condition of indifference can be written as follows:

$$(1-PD)\left[V+Percent\right]-PD\left[disc(t)PLEDGE(t)-SC(t)\right]e^{-r(T-t)}=\beta. \quad (39)$$

Where β is insecurity value.

The condition of indifference of borrower default for credit portfolio of n borrowers can be written as follows:

$$\sum_{i=1}^{n}\frac{V_i}{\sum_{i=1}^{n}V_i}PD_i\left[\sum_{i=1}^{n}disc(t)_i PLEDGE(t)_i-\sum_{i=1}^{n}SC(t)_i\right]e^{-r(T-t)}=$$
$$=(1-\sum_{i=1}^{n}\frac{V_i}{\sum_{i=1}^{n}V_i}PD_i)\left[\sum_{i=1}^{n}V_i+\sum_{i=1}^{n}Percent_i\right] \quad (40)$$
$$0\le PLEDGE(t)_i\le PLEDGE(t)_i \max .$$

With help of (34)-(36) it is possible to build portfolio of trajectories at any moment of time. After building of portfolio of trajectories Monte-Carlo method is used for getting parameters of distribution of the portfolio. Next step is to reach the target portfolio in such a way to satisfy (38). This objective can be achieved in two ways. First is change level of pledge. Equation (38) is an optimization task of bringing the existing portfolio to risk-free by changing the level of collateral. Changing variable is PLEDGEi. Second way is to to expand its portfolio by adding new companies. In second way, the original portfolio is adopted as a portfolio of one borrower. In this case the condition of indifference is given by the following equation:

$$\frac{V}{\sum_{i=1}^{n}V_i+V}PD(t)\left[disc(t)PLEDGE(t)-SC(t)\right]e^{-r(T-t)}+$$
$$+\sum_{i=1}^{n}\frac{V_i}{\sum_{i=1}^{n}V_i+V}PD_i(t)\left[\sum_{i=1}^{n}disc(t)_i PLEDGE(t)_i-\sum_{i=1}^{n}SC(t)_i\right]e^{-r(T-t)}= \quad (41)$$
$$=(1-\frac{V}{\sum_{i=1}^{n}V_i+V}PD(t))\left[V+Percent\right]+$$
$$+(1-\sum_{i=1}^{n}\frac{V_i}{\sum_{i=1}^{n}V_i+V}PD_i(t))\left[\sum_{i=1}^{n}V_i+\sum_{i=1}^{n}Percent_i\right]$$
$$0\le PLEDGE(t)_i\le PLEDGE(t)_i \max .$$

For the case when the portfolio is optimal in terms of the criterion of indifference, addition of a one new borrower is following:

$$\frac{V}{V_n+V}PD(t)\left[disc(t)PLEDGE(t)-SC(t)\right]e^{-r(T-t)}+$$
$$+\frac{V_n}{V_n+V}PD_n(t)\left[disc(t)_n PLEDGE(t)_n-SC(t)_n\right]e^{-r(T-t)}= \quad (42)$$
$$=(1-\frac{V}{V_n+V}PD(t))\left[V+Percent\right]+$$
$$+(1-\frac{V_n}{V_n+V}PD_n(t))\left[V_n+Percent_n\right]$$
$$0\le PLEDGE(t)_n\le PLEDGE(t)_n \max .$$

This is a task of optimization on parameters $PLEDGE_n$ and PD_n. After finding optimal PD, a new task occurs: to find optimal financial indicators of a company which gives such PD. Thus, the problem of finding the optimal borrower comes down to the problem of finding such combinations of financial performance of the borrower, which gives the calculation of the probability of default, which satisfies the criterion of indifference.

5. TASK OF BUILDING PORTRAIT OF PROSPECTIVE BORROWER

In this section the optimization problem of finding the optimal performance of the borrowers is formed. First step is to model financial indicators of bank's borrowers at future moment of time t with help of equations (33)-(36). After that, expectations of borrowers' performance indicators are used for computing future PD of bank's credit portfolio at the moment t. Next step of optimization task is to find optimal PD of potential borrower which makes whole credit portfolio optimal. There are two kinds of limitations in the given case: the borders of the assessed value of the collateral, and the company's plans on increasing the loan portfolio. The optimization task is given in next equation:

$$\frac{V}{V_n+V}PD(t)\left[disc(t)PLEDGE(t)-SC(t)\right]e^{-r(T-t)}+$$
$$+\frac{V_n}{V_n+V}PD_n(t)\left[disc(t)_n PLEDGE(t)_n-SC(t)_n\right]e^{-r(T-t)}\to \quad (43)$$
$$\to(1-\frac{V}{V_n+V}PD(t))\left[V+Percent\right]+$$
$$+(1-\frac{V_n}{V_n+V}PD_n(t))\left[V_n+Percent_n\right]$$
$$0\le V(t)_n\le V(t)_n \max$$
$$0\le PLEDGE(t)_n\le PLEDGE(t)_n \max .$$

After optimal PD of potential borrower is defined, next step is to find all possible combinations of financial indicators of a borrower. To do that, analyst has to solve next optimization task:

$$\frac{e^{\theta}}{1+e^{\theta}}\to PD$$
$$\theta=-1{,}122-0.272x_1-4.731x_2+5.702x_3+0.002x_4+0.119x_5 \quad (44)$$
$$0<x_i<x_{i\max}.$$

Here x_i is respectively one of coefficients which is described in (37). Solving this optimization task gives several possible combinations of potential borrower's performance indicators which can be used as a benchmark for new clients of a bank.

6. CONCLUSIONS

Described model gives instruments of forecasting credit portfolio quality at future moments of time, shows necessary preventing activities when condition of bank's indifference is broken and draws parameters and financial indicators of potential borrower. Thus, total risk of the portfolio is monitored and is stored at acceptable for the bank's level.

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